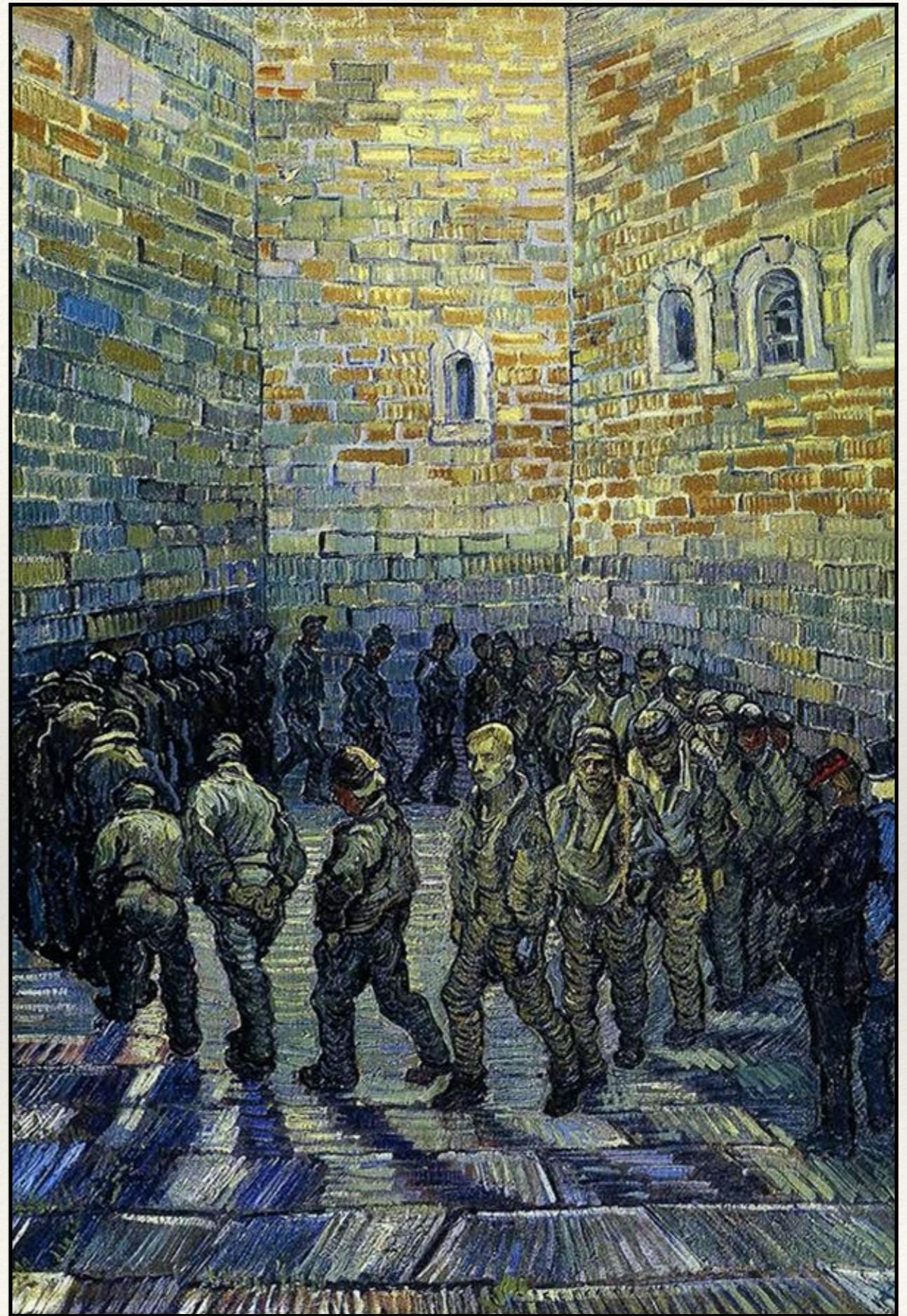

A Two-dimensional Theory of Proof Beyond a Reasonable Doubt

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The Proposal

Req 1: the guilt probability on the evidence should be suitably high, for example, 95%

high guilt
probability
threshold

Dimension 1: guilt probability

$\text{Pr}(\text{Guilty} \mid \text{Evidence})$

Req 2: the body of evidence presented at trial should be as inclusive as reasonably possible

reasonable
evidential weight

Dimension 2: evidential weight

how much evidence is
presented at trial

Illustration

Scenario A

Two witnesses testify against the defendant and survive vigorous cross-examination by the defense

- ✓ Req 1: high guilt probability
- ✓ Req 2: weight requirement

Scenario B

The same as in scenario A *except that the prosecutor does not call two other witnesses present during the crime*

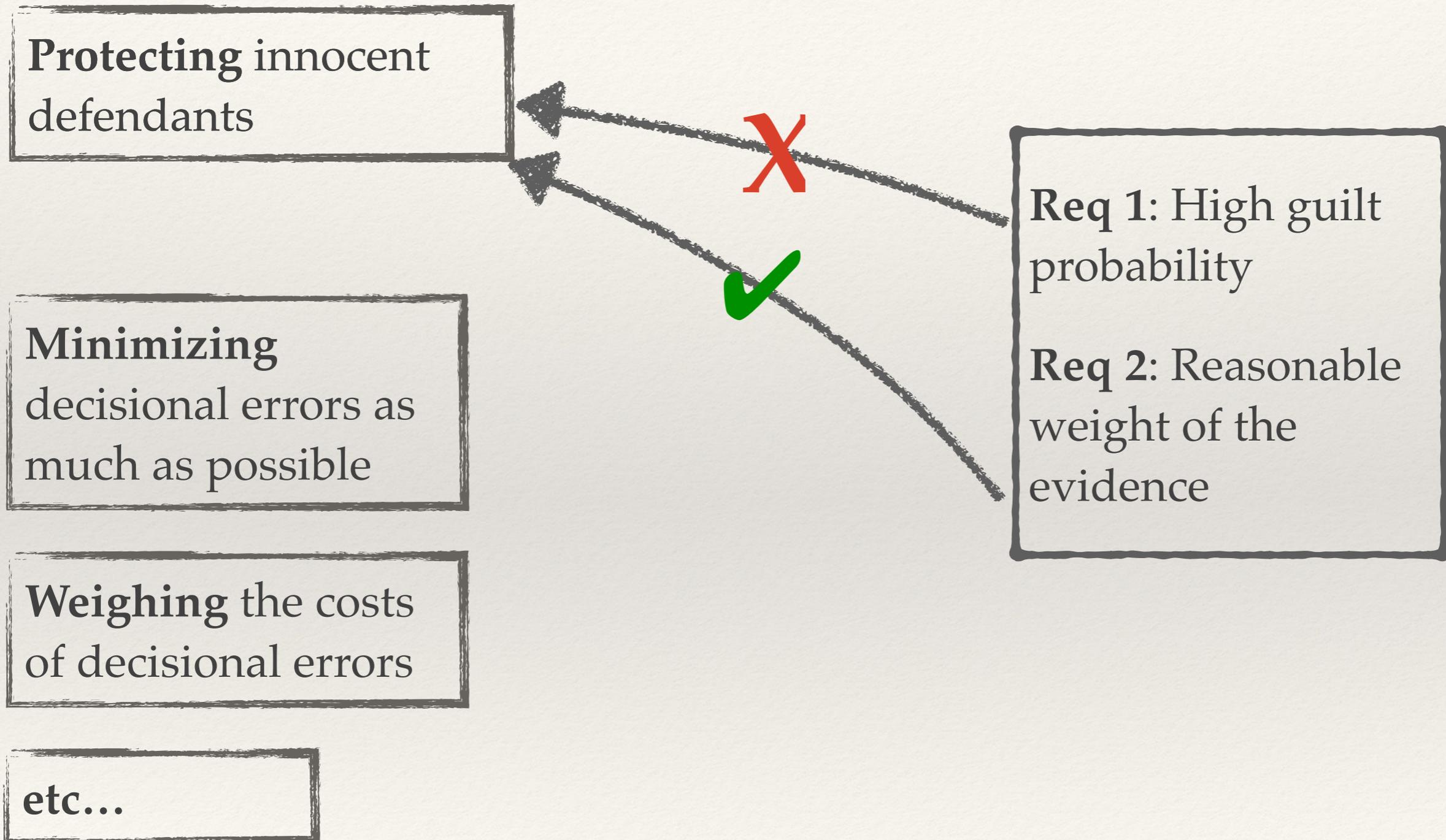
- ✓ Req 1: high guilt probability
- ✗ Req 2: weight requirement

The evidence presented at trial is the same in the two scenarios, so *the guilt probability on the evidence must be the same*. The difference between the scenarios must lie in whether the weight requirement is met

Why Not Only

High Guilt Probability Threshold?

Functions of the Burden of Proof



Why a High (Probability) Threshold
Does *Not* Protect Innocent Defendants

Guilt Probability v. Error Risk

$\text{Pr}(\text{Guilty} \mid \text{Evidence})$

guilt probability
on the evidence

$\text{Pr}(\text{Conviction} \mid \text{Innocent})$

(conditional) risk
of mistaken
conviction

The lower the error risk,
the better the protection
innocent defendants enjoy

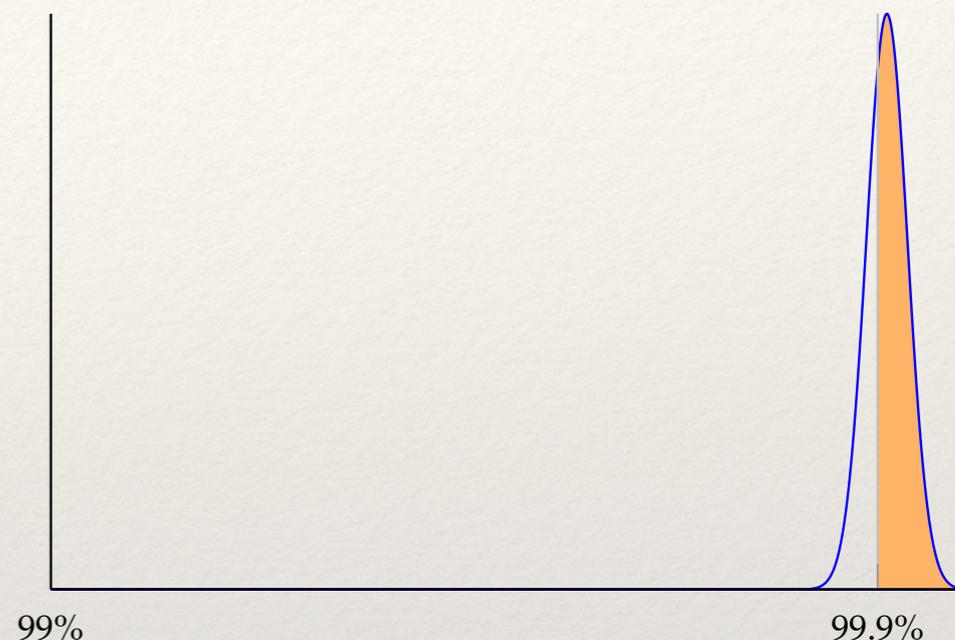
Does a *High Threshold* Ensure a *Low Risk*?

- ❖ the guilt probability on the evidence should be high, for example, 99%

high
threshold

- ❖ the risk of mistaken conviction should be low, for example, 1%

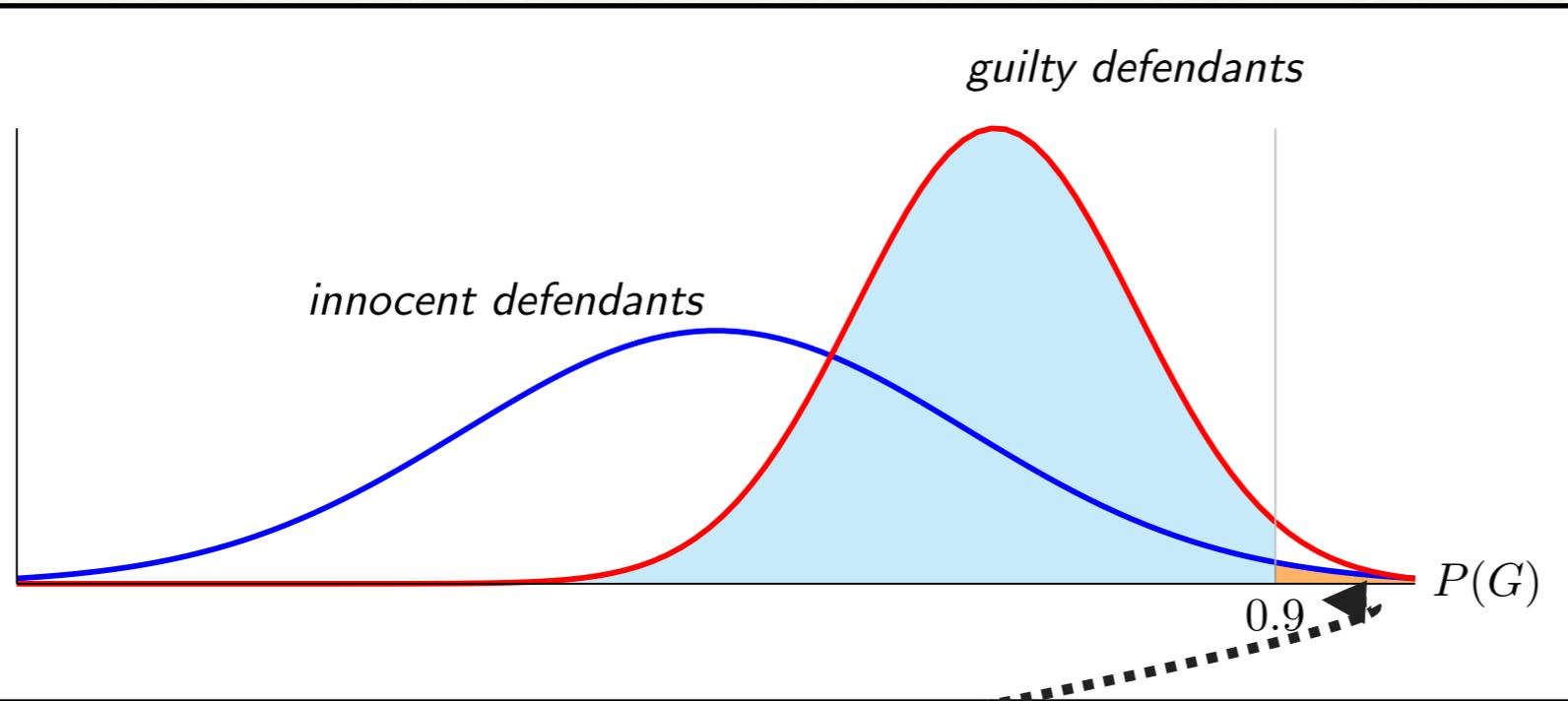
low risk



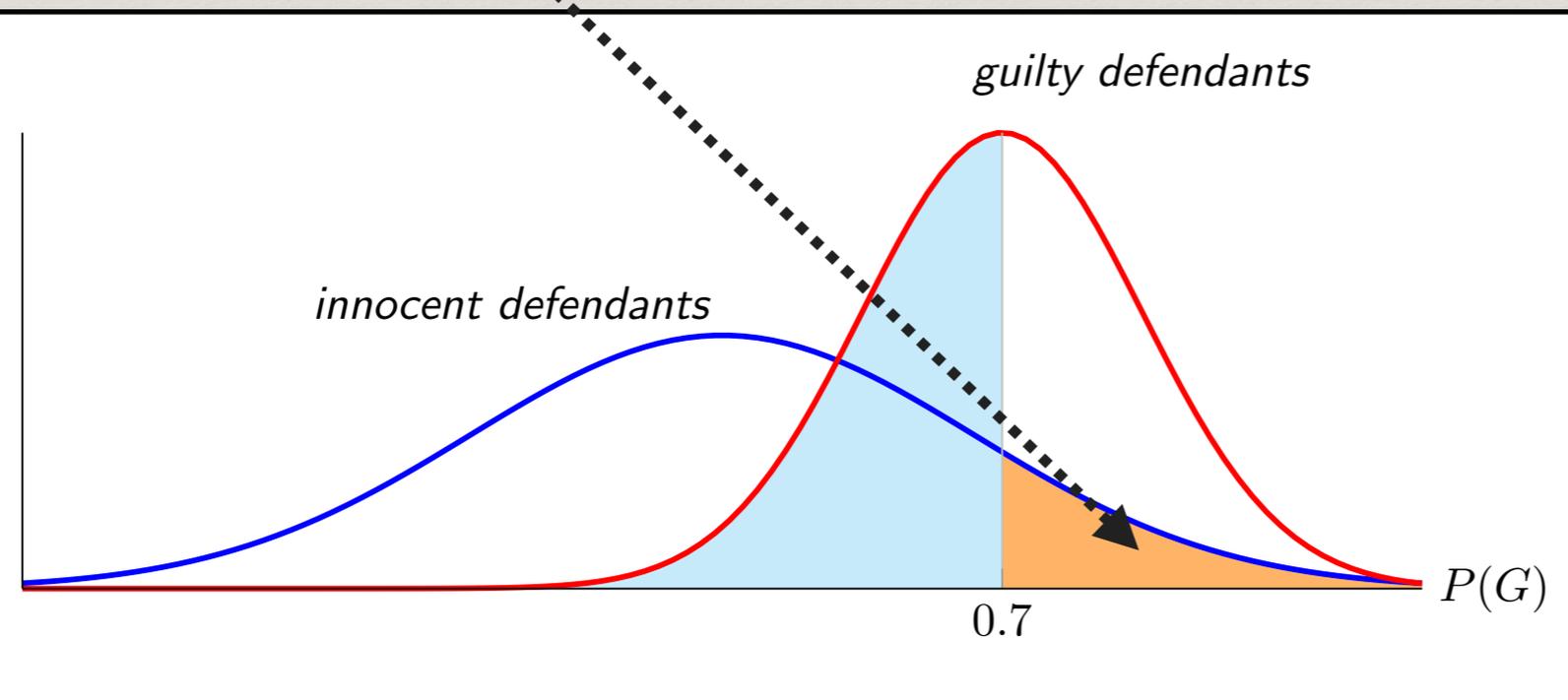
GUILT PROBABILITY

Given a certain distribution of innocent defendants, the risk of mistaken conviction could still be above 50% even with a 99.9% threshold

Higher Threshold / Lower Risk - *Right?*

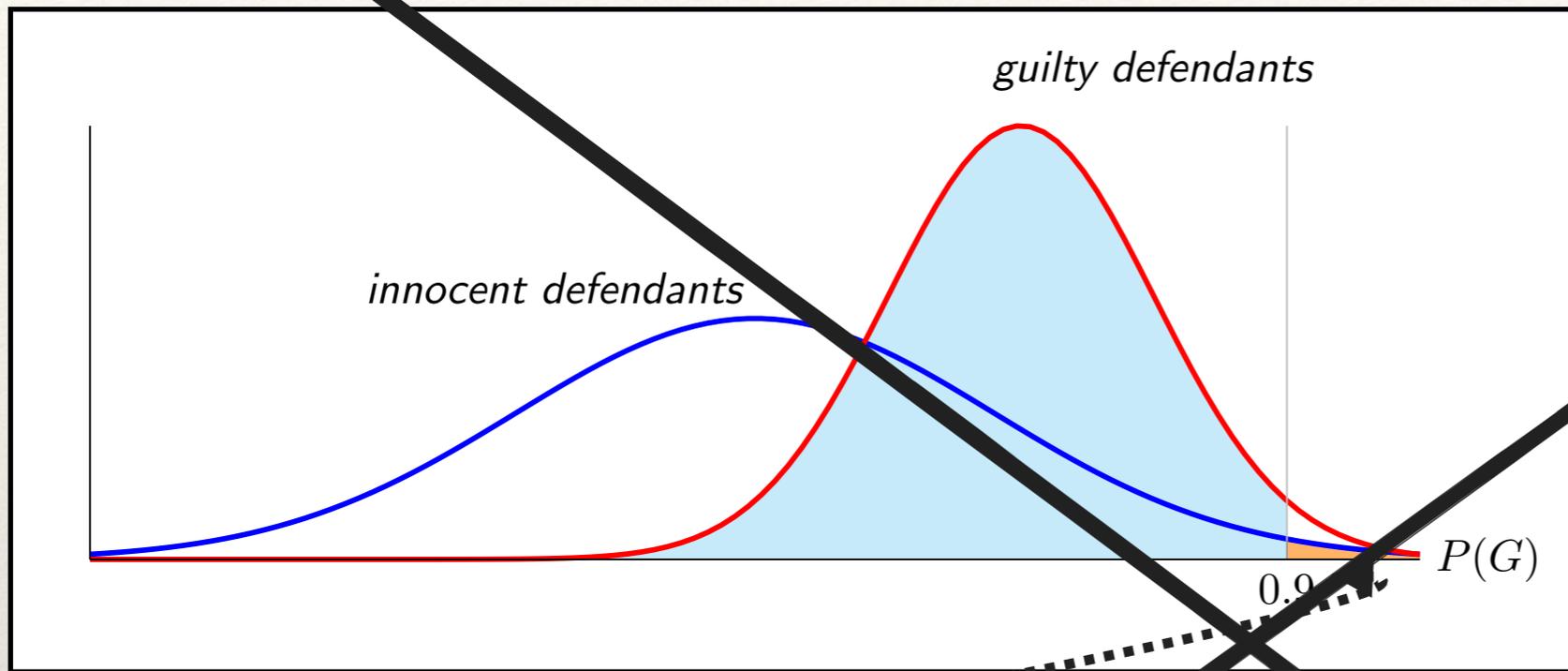


Risk of mistaken conviction

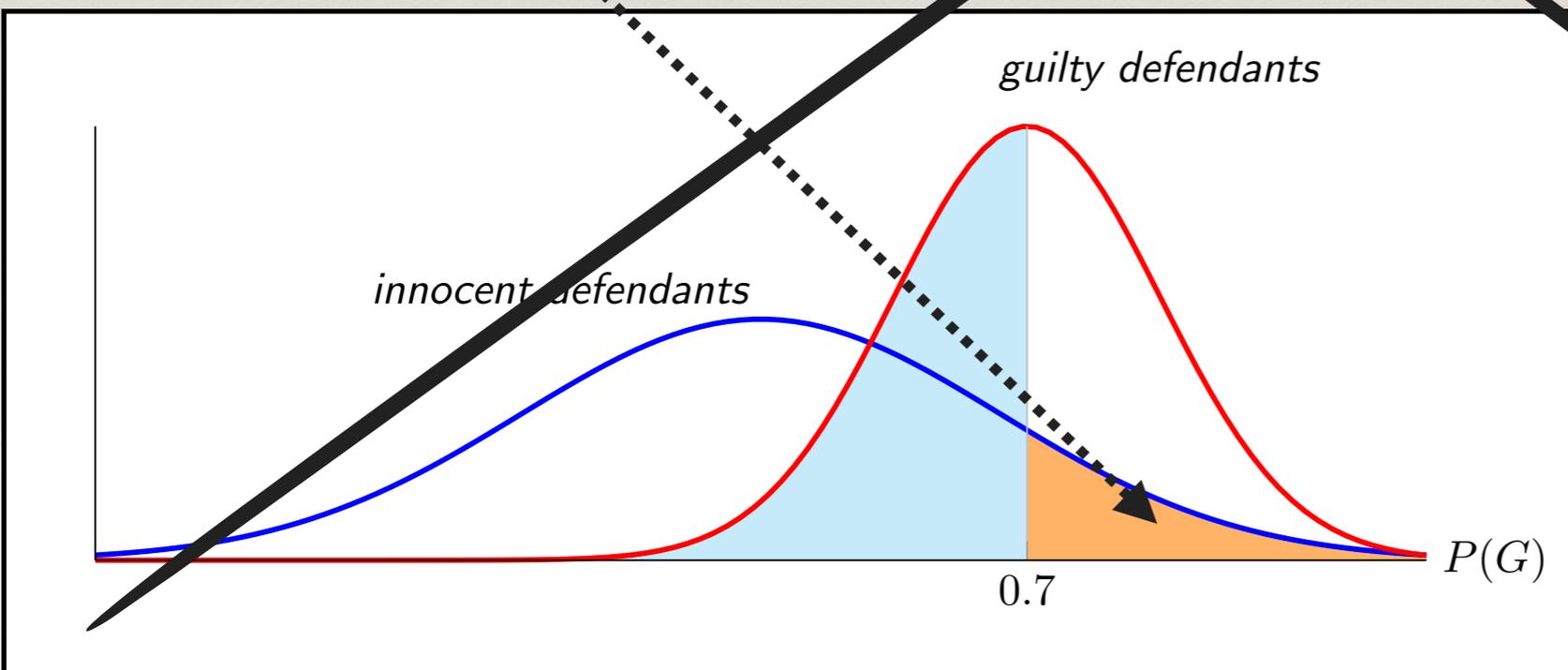


It *would seem* that the higher (lower) the probability threshold, the lower (higher) the risk of mistaken conviction

Higher Threshold / Lower Risk - *Right?*

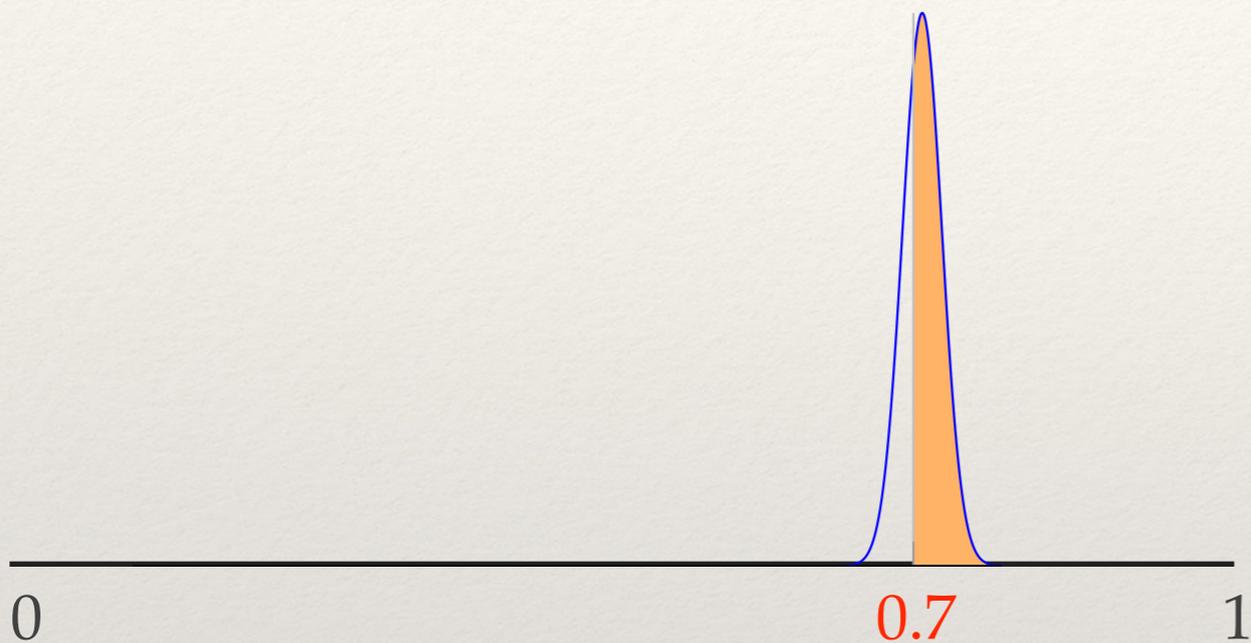


Risk of mistaken conviction

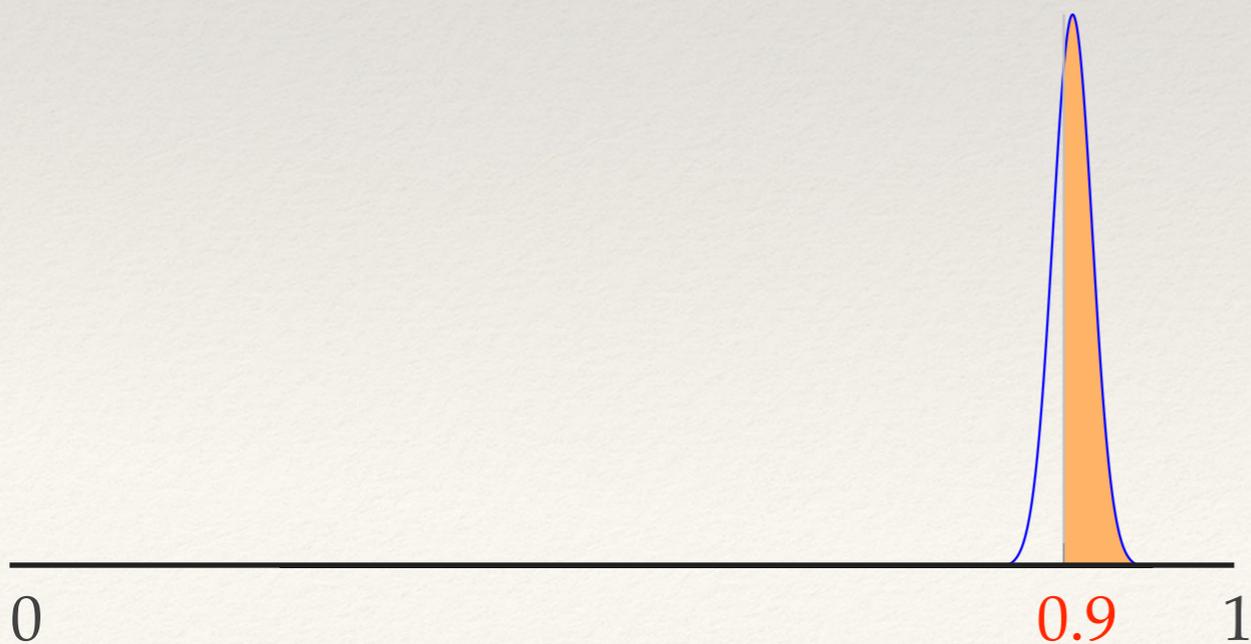


It *would seem* that the higher (lower) the probability threshold, the lower (higher) the risk of mistaken conviction

Feedback Mechanisms



Given a higher threshold, prosecutors will bring to trial only those defendants who stand a good chance of being convicted (*feedback mechanisms*)



The distribution of innocent defendants will thus move along with the higher threshold, so the error risk will be constant

What Does a High Threshold Do?

~~Protecting innocent
defendants~~

Minimizing
decisional errors as
much as possible

Weighing the costs
of decisional errors

etc...

How are innocent
defendants protected
if a high threshold
does not protect them?

Why the Weight Requirement Protects Innocent Defendants

Probability Changes and Diagnosticity

The presentation of new evidence at trial will cause the guilt probability estimate of the defendant to increase \uparrow (*incriminating*) or decrease \downarrow (*exculpatory*)

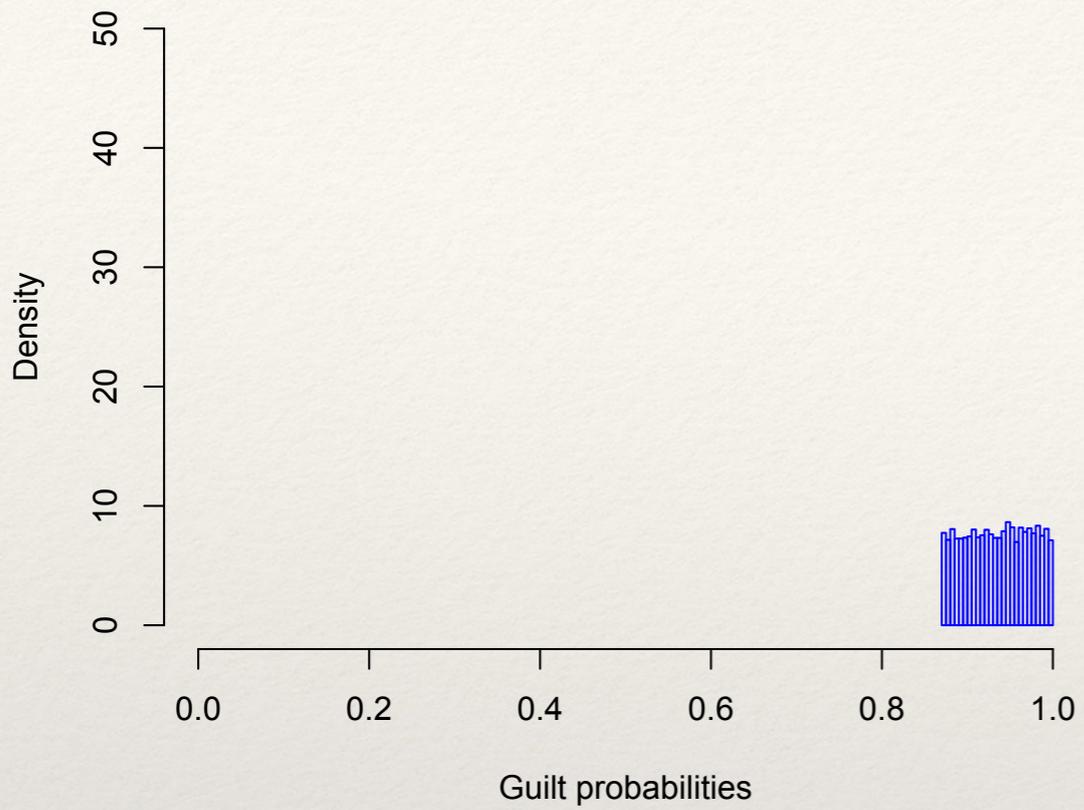
The probability changes \uparrow and \downarrow have diagnosticity d , provided

$$Fr(\uparrow | G) > d\%$$

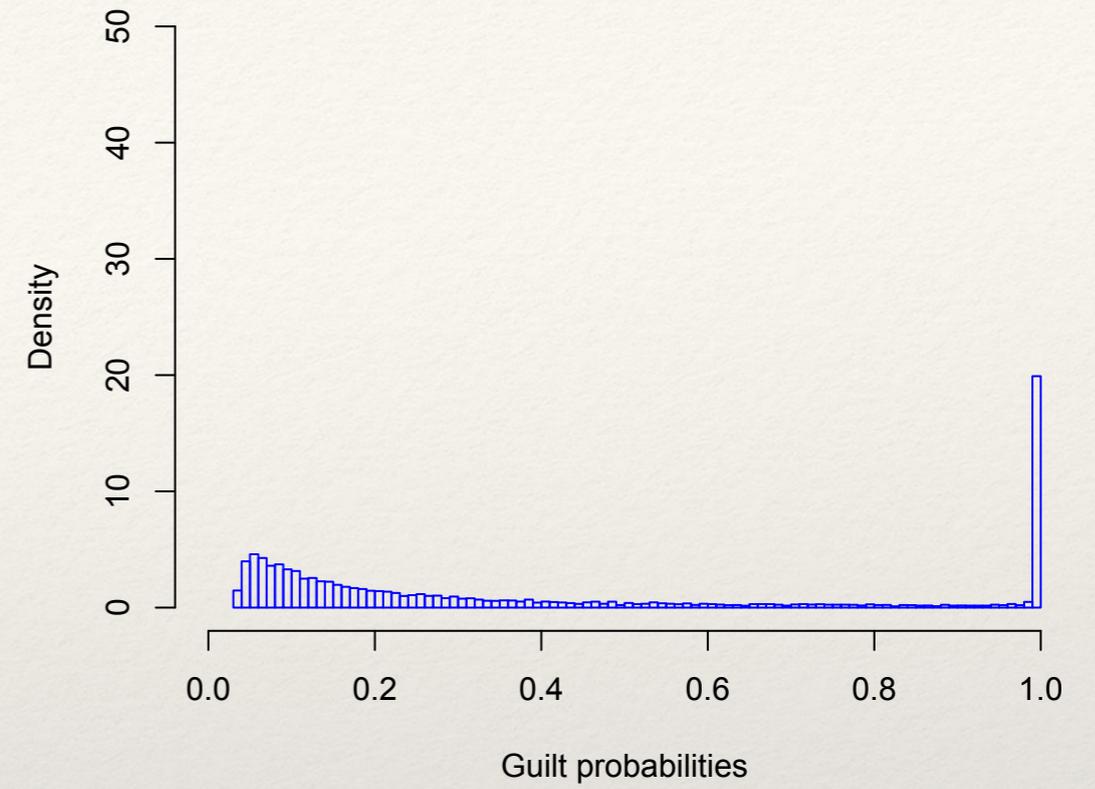
and

$$Fr(\downarrow | I) > d\%$$

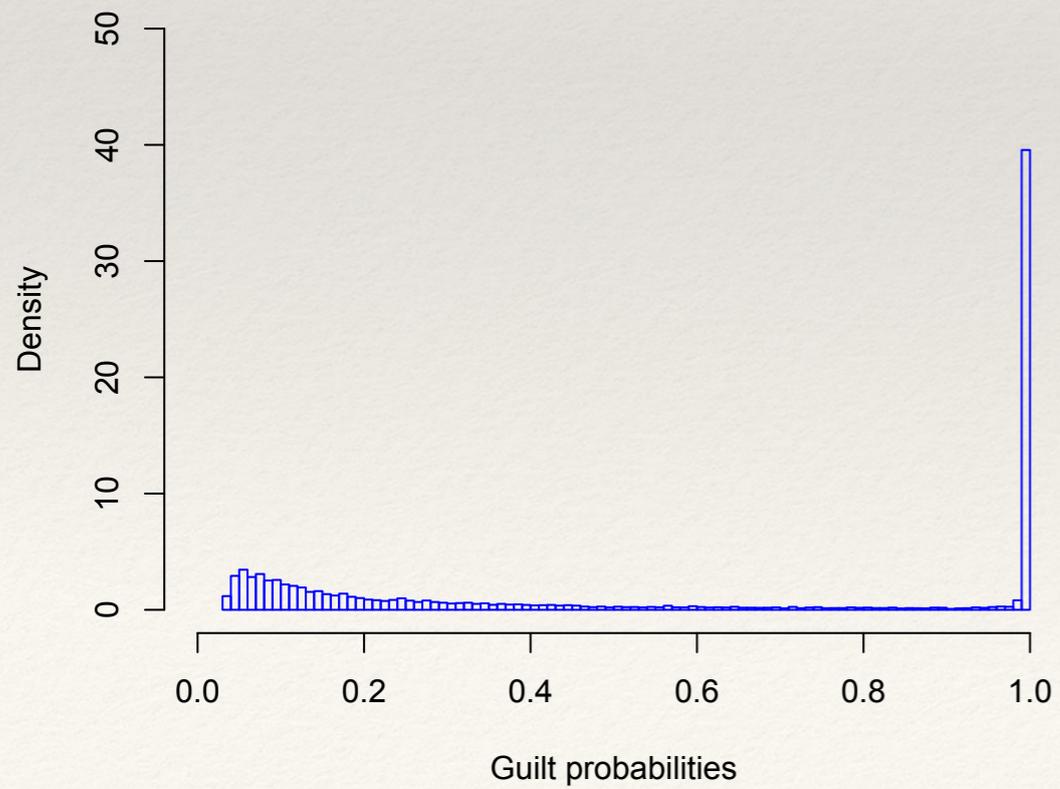
Initial distribution of innocent defendants



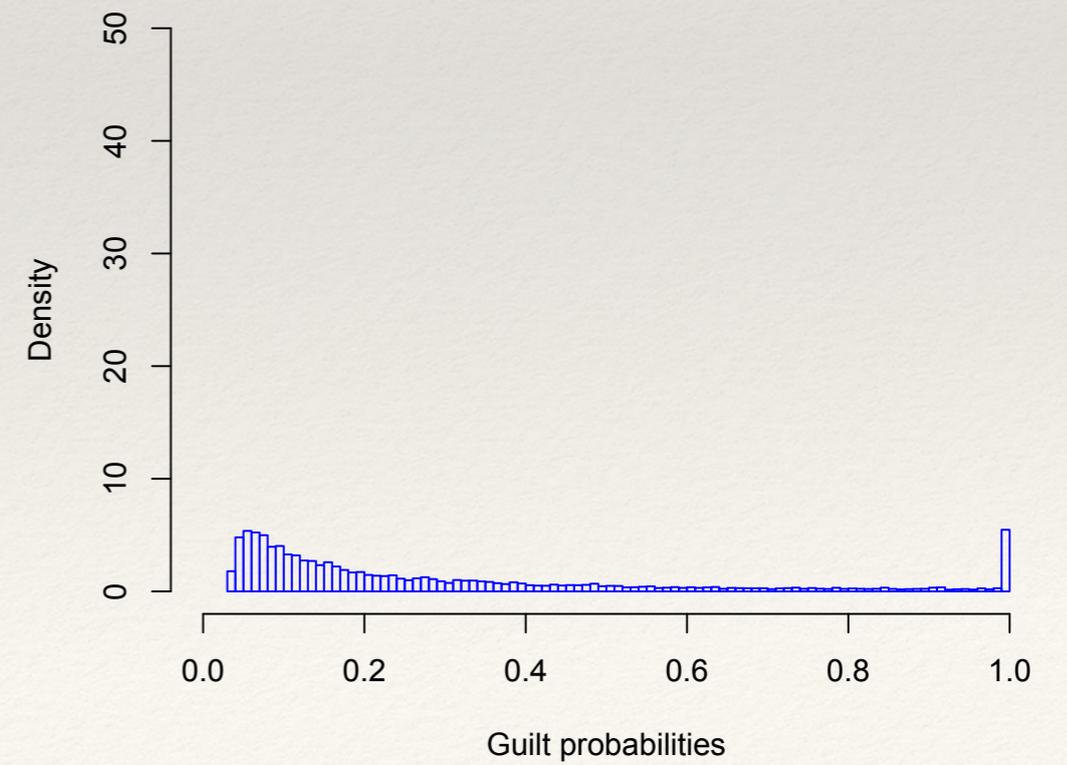
80 percent diagnosticity



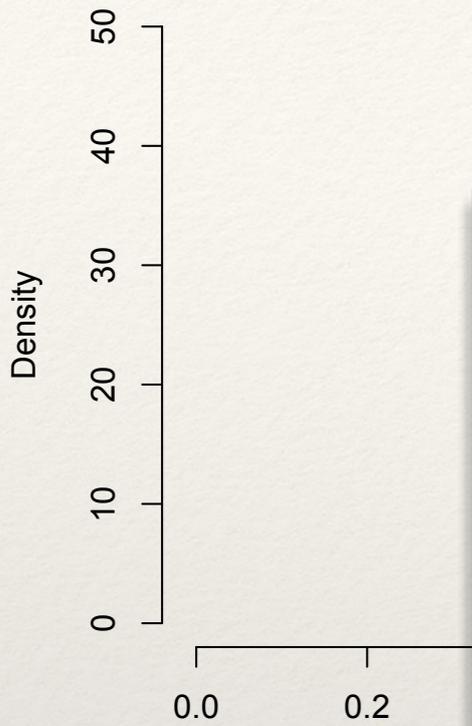
60 percent diagnosticity



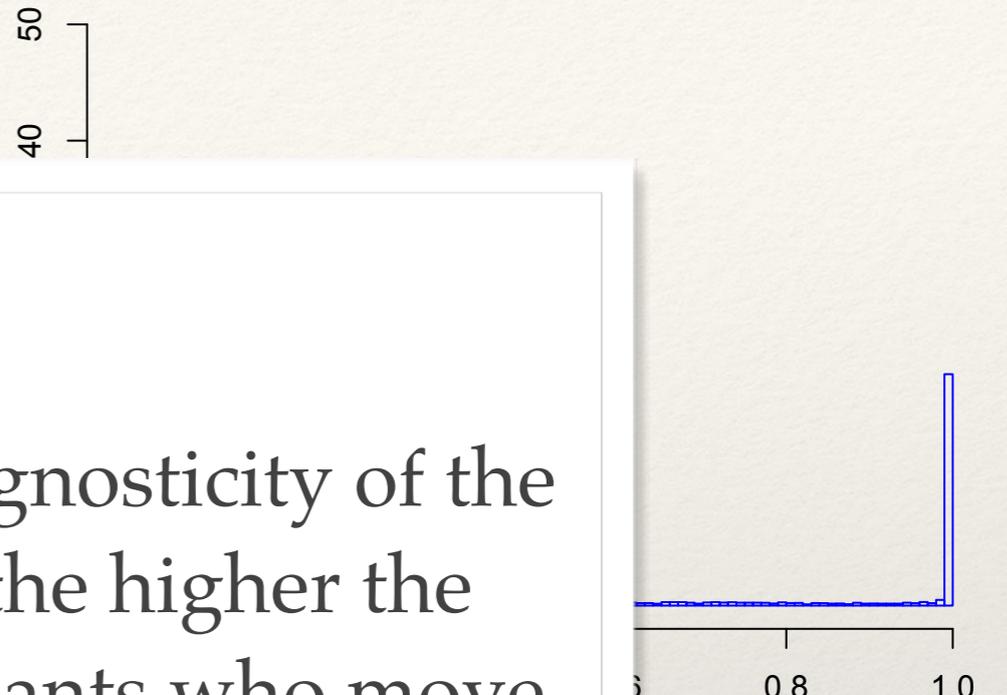
95 percent diagnosticity



Initial distribution of innocent defendants

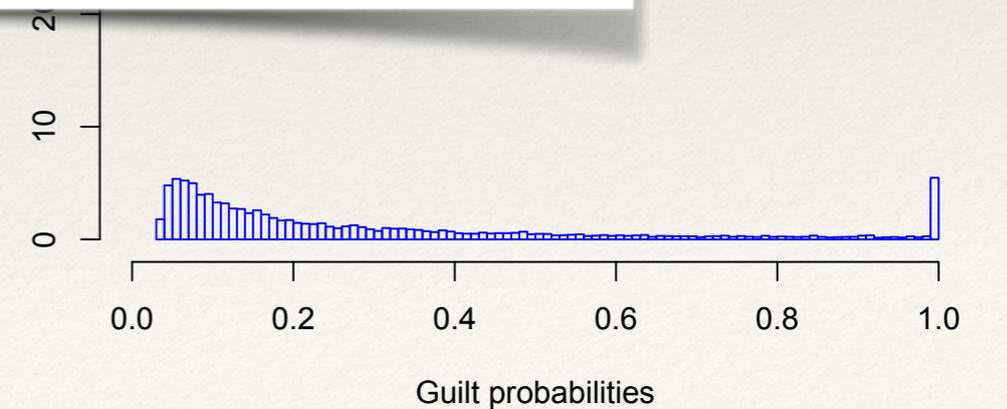
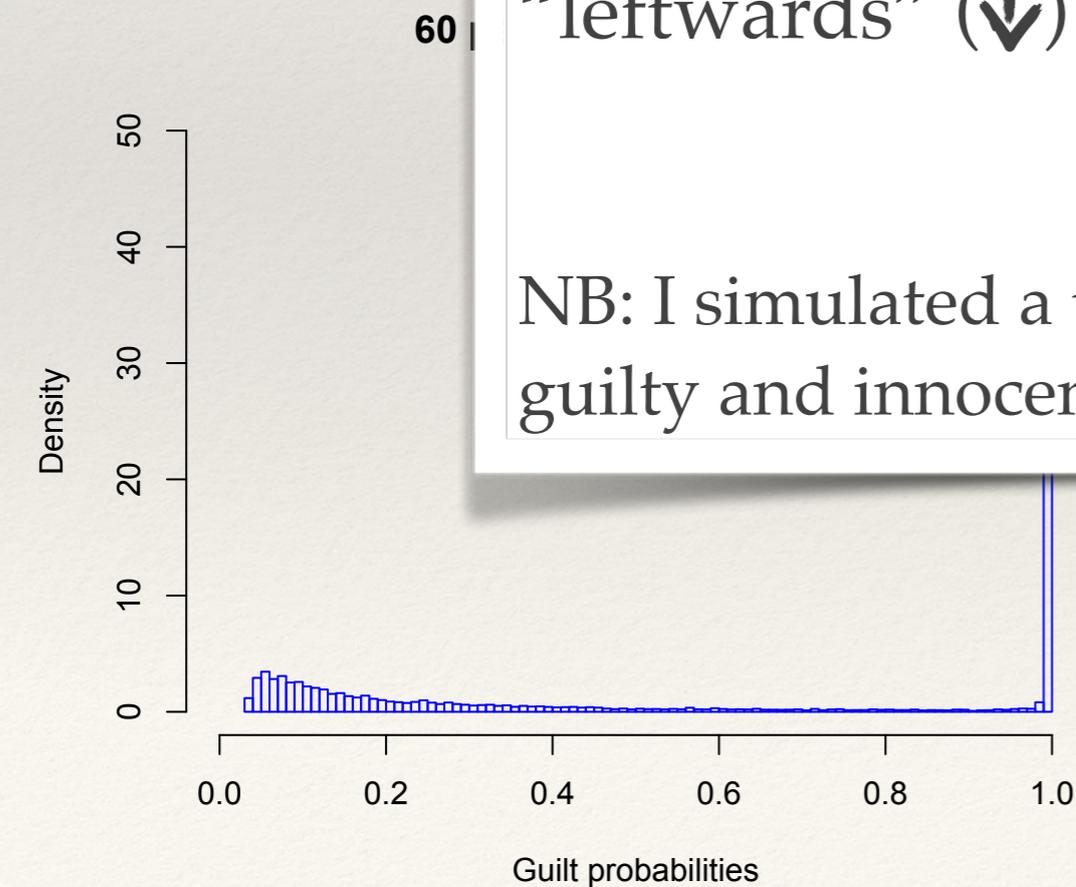


80 percent diagnosticity

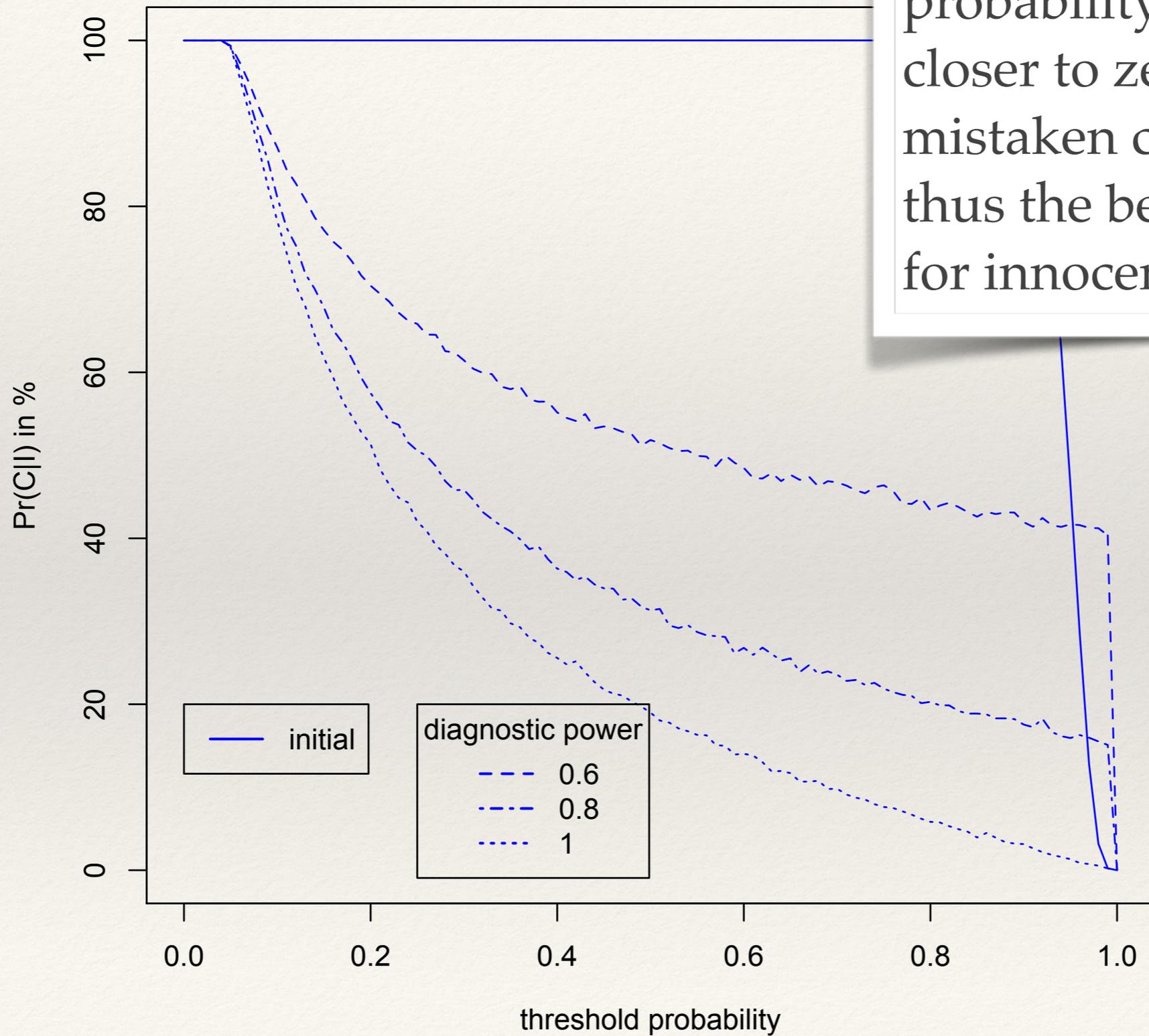


Conclusion: The higher the diagnosticity of the probability changes \downarrow at trial, the higher the percentage of innocent defendants who move “leftwards” (\downarrow)

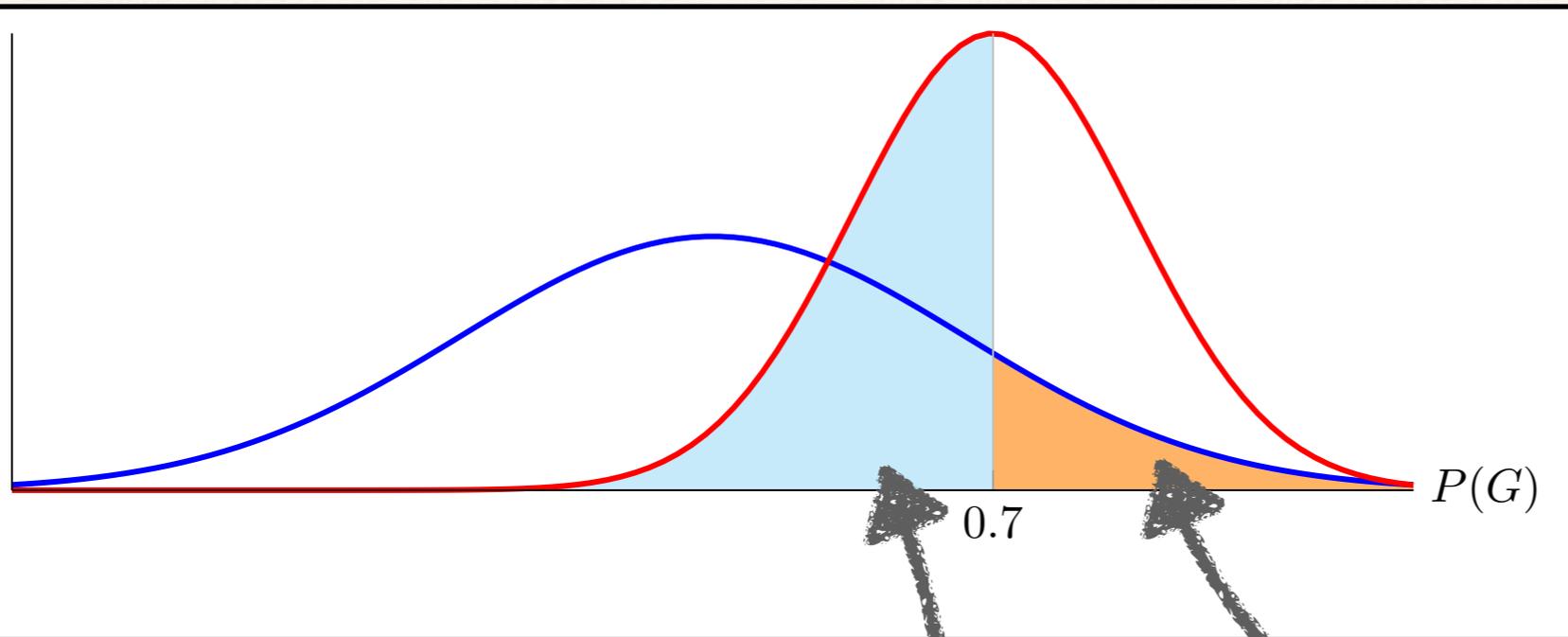
NB: I simulated a trial system with 20,000 defendants, guilty and innocent, using the language R



Conclusion: The higher the diagnosticity of the probability changes \downarrow , the closer to zero the risk of mistaken conviction, and thus the better the projection for innocent defendants



Two Error Risks



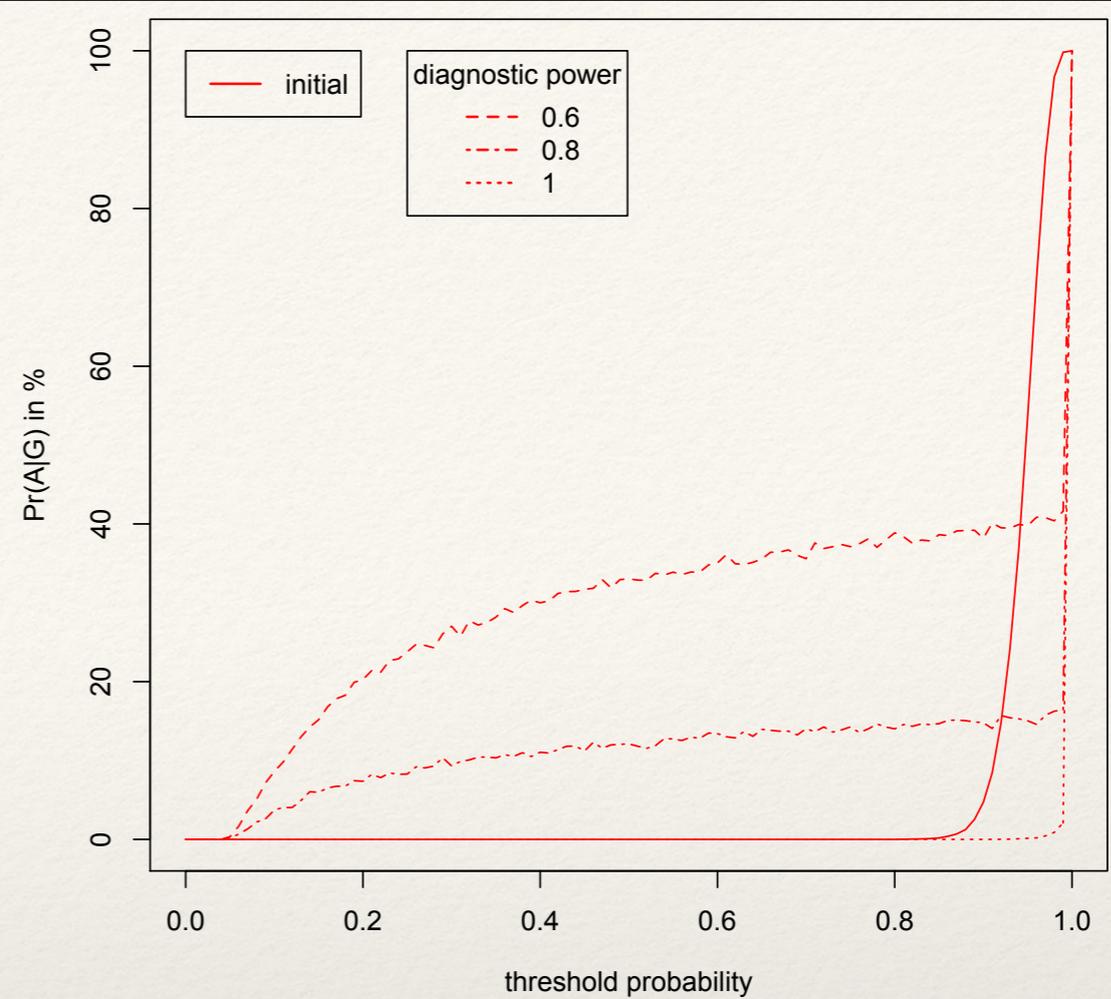
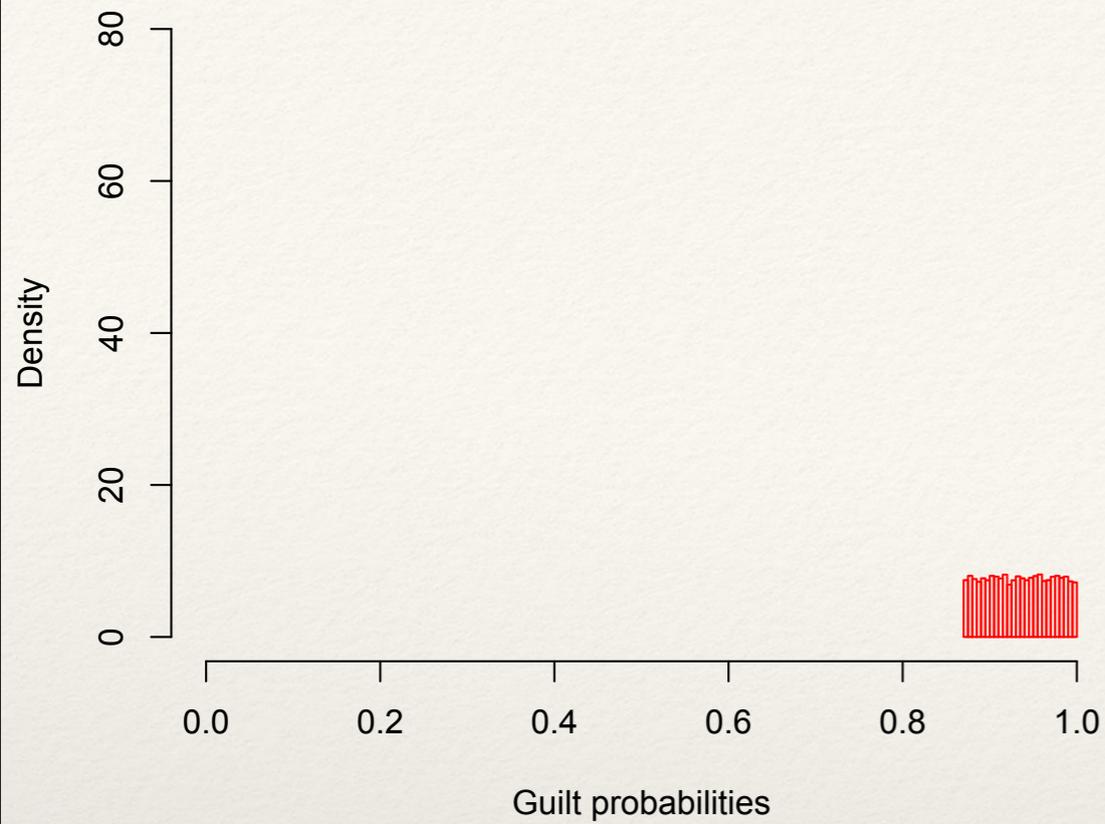
$\Pr(\text{Conviction} \mid \text{Innocent})$

(conditional)
risk of mistaken
conviction

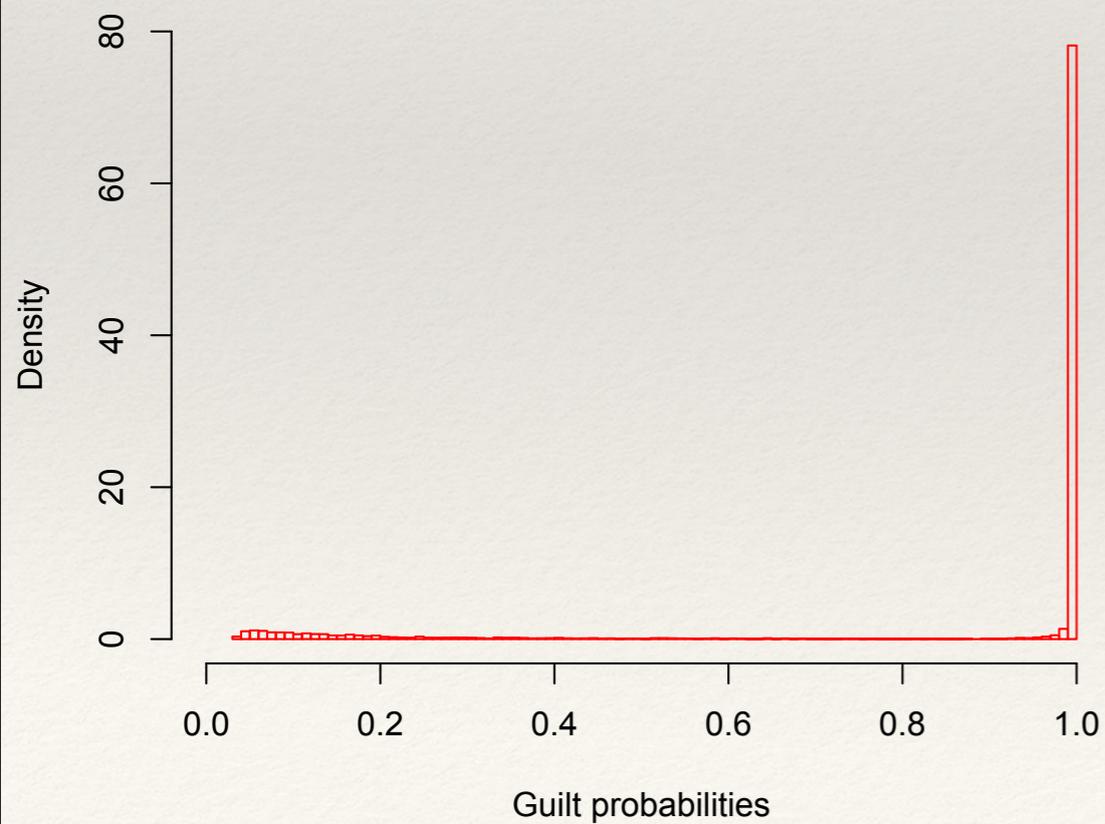
$\Pr(\text{Acquittal} \mid \text{Guilty})$

(conditional) risk
of mistaken
acquittal

Initial distribution of guilty defendants



80 percent diagnosticity



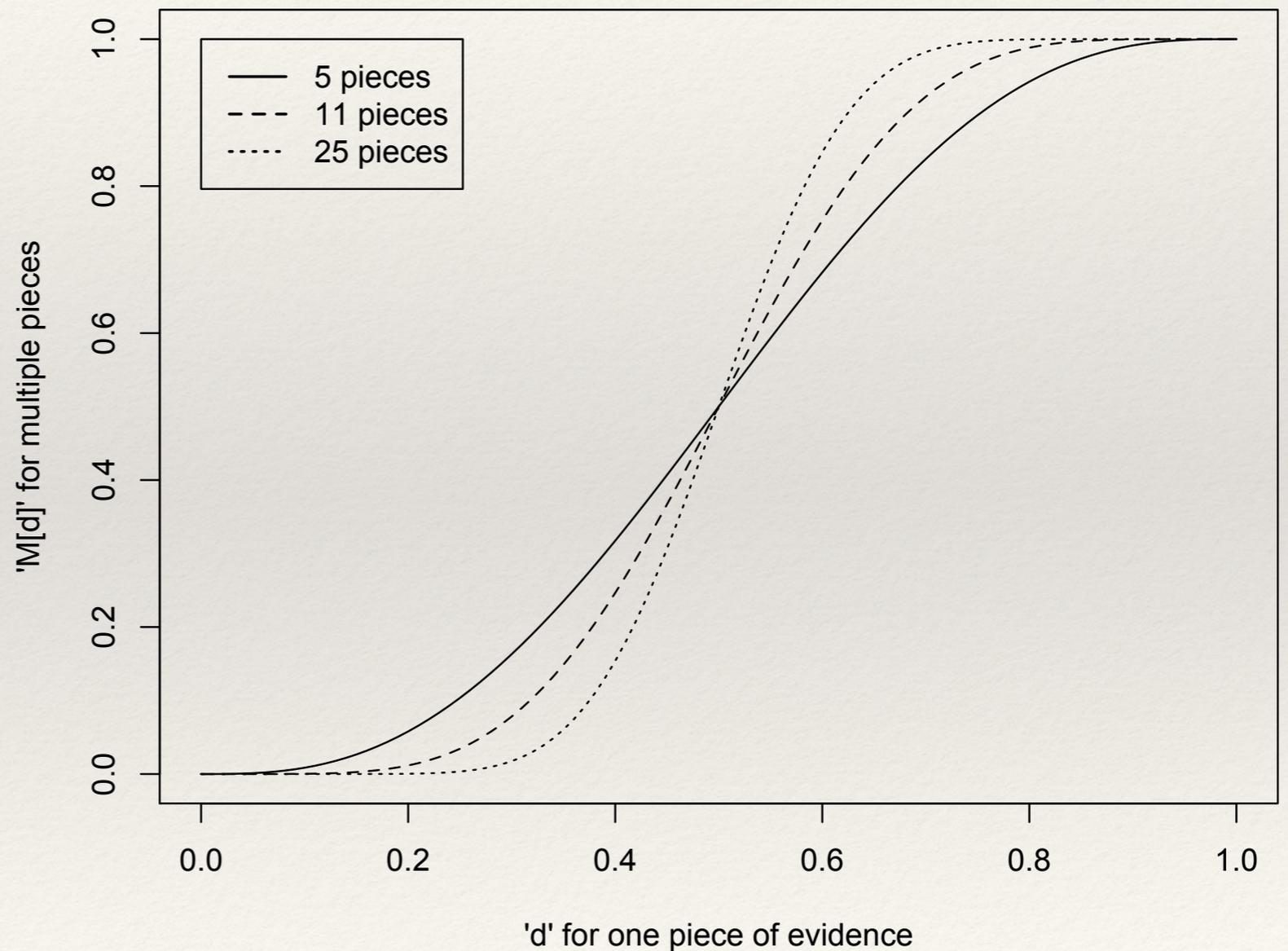
Conclusion: The higher the diagnosticity of the probability changes \uparrow , the closer to zero the risk of mistaken acquittal

How to Improve Diagnosticity?

The more evidence presented at trial (*weight*), the higher the diagnosticity *provided the evidence is better than chance* [$d > 50\%$]

(Note the similarity with Condorcet's Jury Theorem)

Diagnosticity and pieces of evidence



Weight requirement: the body of evidence presented at trial should be as inclusive as reasonably possible

*more evidence
(incriminating
and exculpatory)*

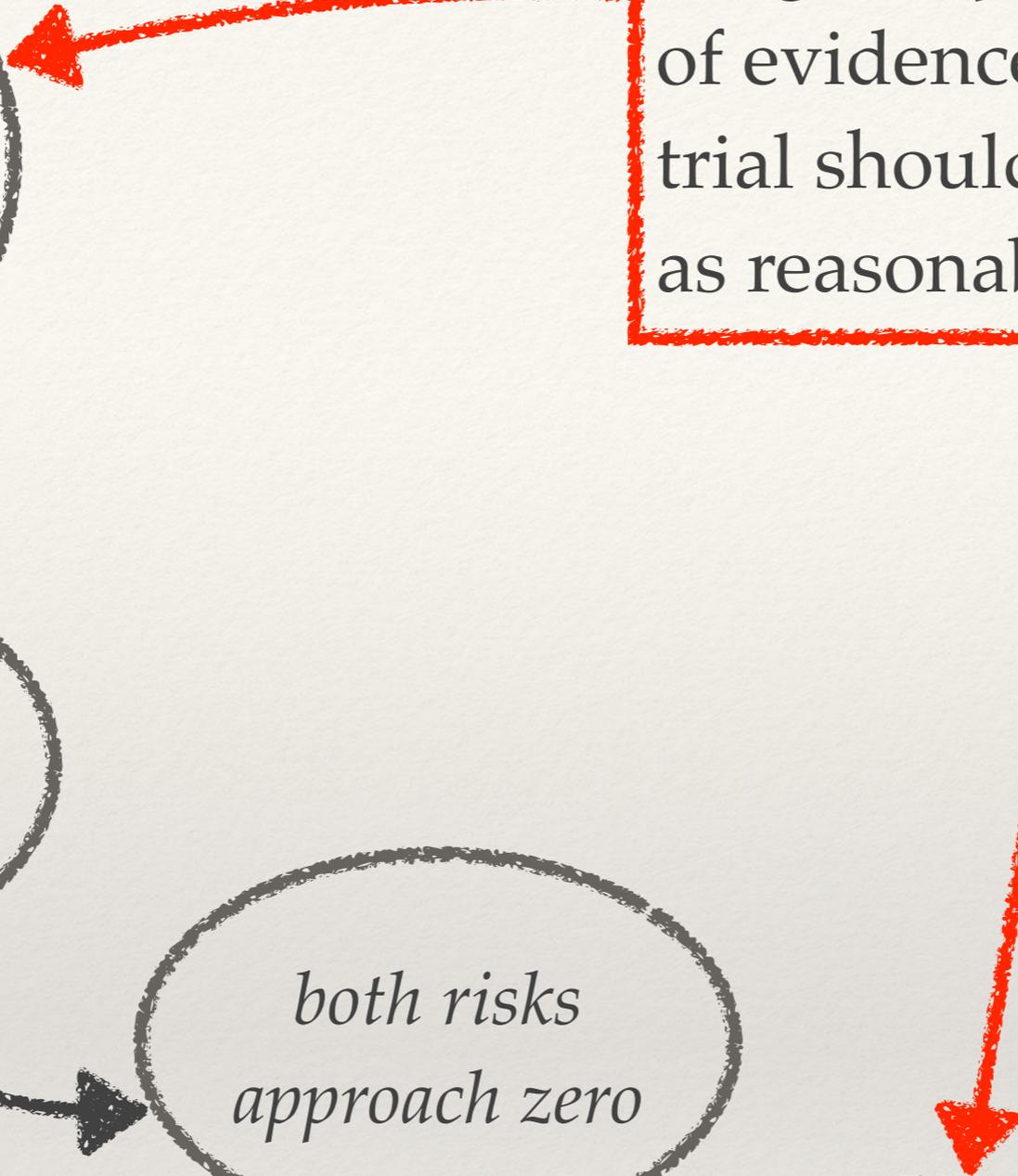
*higher
diagnostic
power*

*both risks
approach zero*

*higher threshold
probability*

*better
protection for
innocent
defendants*

*false
acquittals are
more often
avoided*



Weight requirement: the body of evidence in a trial as r

NB: The weight requirement does not determine a fix threshold, but demands that *as much evidence as reasonably possible* be presented, and thus, can keep error risks *as low as reasonably possible*

more evidence
(incriminating
and exculpatory)

higher
diagnostic
power

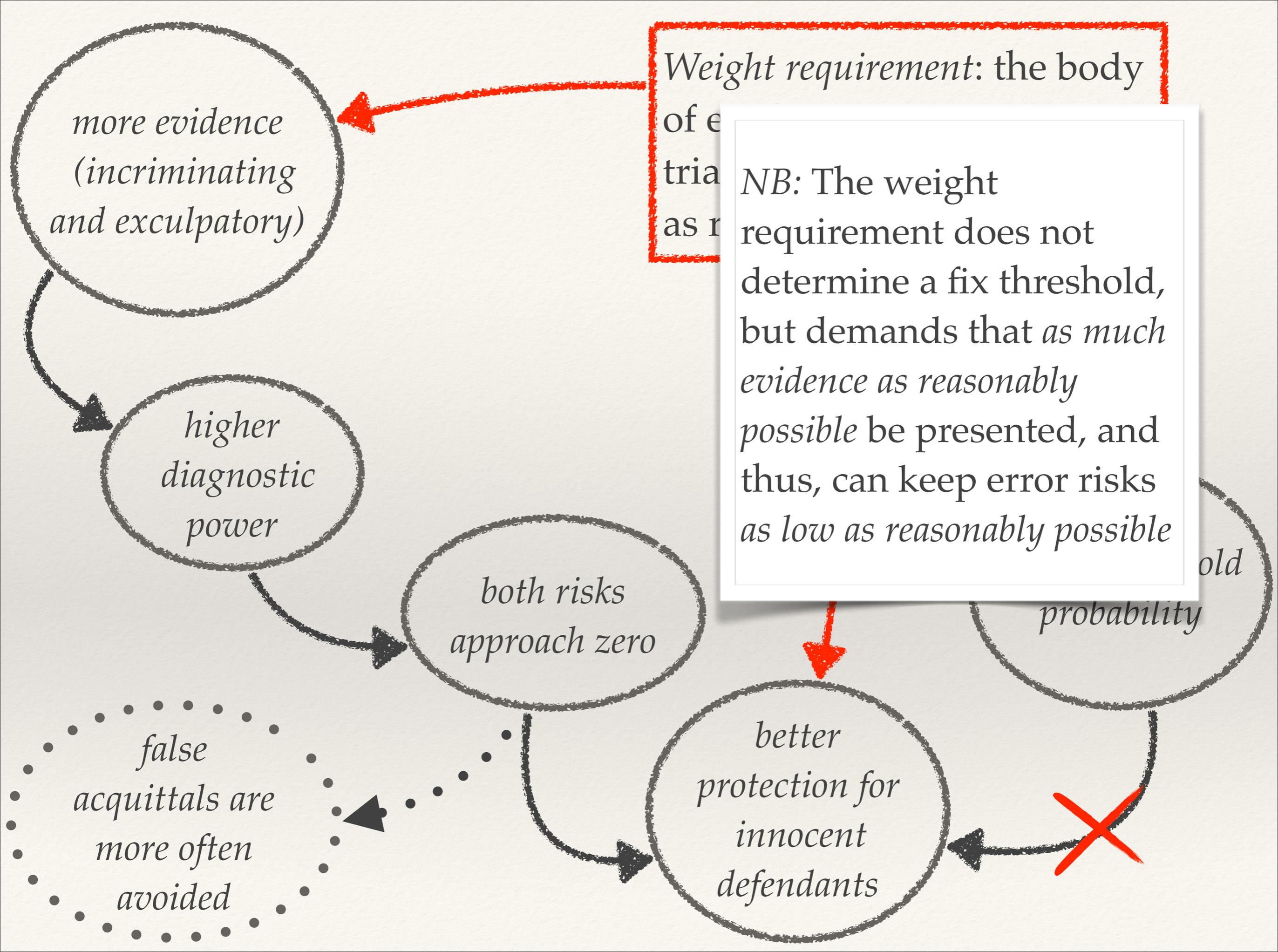
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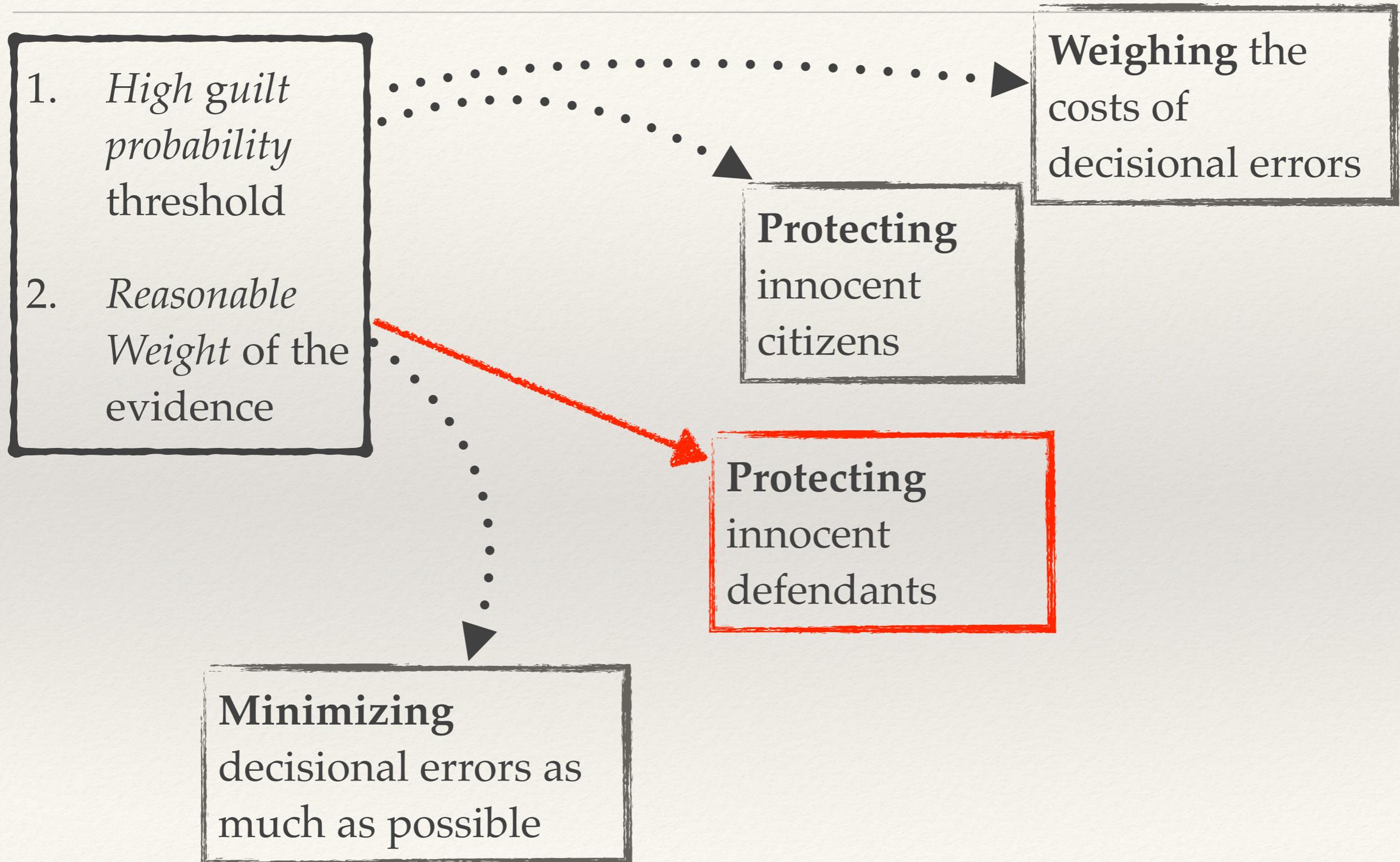
better
protection for
innocent
defendants

probability

old



The *Two-dimensional Theory*



Thank you!